Classic Transmission Lines

This text is an elaboration on "Classic Transmission Line Enclosure Tables" by Martin J King. In that paper a formula and tabulated function values are given. By finding approximations to the tabulated functions we can find some interesting results:

1. we can investigate how parameters affect the end result (e.g. optimize) and
2. we can quickly calculate a parameter set to start from when we use Martin King's Mathcad worksheets.

The formulas, below, are all copy-past from a Mathcad worksheet of mine. The approximations were created using MS Excel trend line function.

Theory

Definitions

- ρ = density of air
- c = speed of sound in air
- BL = magnetic force parameter
- Re = voice coil electric resistance
- Sd = driver area
- S0 = cross sectional area of the driver end of the transmission line (TL).
- SL = cross sectional area of the exit end of the TL
- Leff = the effective length of the TL.
- Lact = actual length of the TL. Lact = Leff – end correction.

M J King gives the following equation in his text,

$$S_0 := \rho \cdot c \cdot S_d^{\,2} \cdot D_Z \cdot D_R \cdot \frac{R_e}{BL^2}$$

We start by approximating DR, based on tabulated values, table 1.

Table 1. DR as a function of Qts, DR1 & DR2 and relative errors

Qts	DR	DR1	error %	DR2	error %
0.2	**0.1858**	0.1839	1.0	0.1860	-0.1
0.3	**0.1313**	0.1281	2.5	0.1303	0.8
0.4	**0.0950**	0.0990	-4.1	0.0964	-1.4
0.5	**0.0788**	0.0812	-2.9	0.0780	1.0
0.6	**0.0688**	0.0690	-0.2	0.0688	0.0
0.7	**0.0625**	0.0601	4.0	0.0625	0.0

$$D_{R1}(Q_{ts}) := 0.0437 \cdot Q_{ts}^{\,-0.893}$$

$$D_{R2}(Q_{ts}) := -1.0574 \cdot Q_{ts}^{\,3} + 2.0457 \cdot Q_{ts}^{\,2} - 1.3797 \cdot Q_{ts} + 0.388€$$

The relative errors are given in table 1. Especially for DR2, the error is negligible.

A good approximation to shape function DZ, table 3, is

$$D_Z(\alpha, f_B) := (-0.359 \cdot \ln(\alpha) + 0.9839) \cdot f_B$$

The relative error is given in table 2. We see that for small alfa, the error is very small.

Table 2. Relative error of DZ approximation

α	10	5	3	2	1	0.5	0.333	0.2	0.1
error, [%]	39,6	-4.4	-6.2	-4.6	-1.3	0.6	0.9	0.8	0.7

Table 3. Shape function DZ as a function of SL/S0 = α and frequency

SL/S0	20	25	30	35	40	45	50	55	60	65	70
10.00	4.39	5.49	6.59	7.69	8.78	9.88	10.96	12.08	13.18	14.27	15.37
5.00	7.77	9.71	11.65	13.59	15.53	17.47	19.42	21.36	23.30	25.24	27.18
3.00	11.06	13.82	16.59	19.35	22.12	24.88	27.65	30.41	33.18	35.94	38.71
2.00	14.03	17.53	21.04	24.54	28.05	31.57	36.06	38.67	42.08	45.58	49.09
1.00	19.43	24.29	29.14	34.00	38.86	43.72	48.57	53.43	58.29	63.14	68.00
0.50	24.79	30.99	37.19	43.39	49.59	55.79	61.99	68.18	74.38	80.58	86.78
0.33	27.82	34.78	41.73	48.69	55.64	62.60	69.55	76.51	83.46	90.42	97.37
0.20	31.49	39.37	47.24	55.12	62.00	70.86	78.74	86.61	94.48	102.36	110.23
0.10	36.48	45.60	54.72	63.84	72.96	82.09	91.21	100.33	109.45	118.57	127.69

The following ratios are of interest: $\sigma_0 := \frac{S_0}{S_d}$, $\alpha := \frac{S_L}{S_0}$

With the expressions for DR and DZ, we can now calculate

$$\sigma_0(\alpha, f_B, Q_{ts}) := \rho \cdot c_{air} \cdot S_d \cdot D_Z(\alpha, f_B) \cdot D_{R2}(Q_{ts}) \cdot \frac{R_e}{BL^2}, \quad \text{and}$$

$$S_0 := \sigma_0(\alpha, f_B, Q_{ts}) \cdot S_d \qquad S_0(\alpha, f_B, Q_{ts}) := \sigma_0(\alpha, f_B, Q_{ts}) \cdot S_d$$

$$S_L := \alpha \cdot S_0 \qquad S_L(\alpha, f_B, Q_{ts}) := \alpha \cdot \sigma_0(\alpha, f_B, Q_{ts}) \cdot S_d$$

The effective length Leff is also given in a table. We first convert to meter, then we get, table 4.

Table 4. TL effective length as a function of frequency and shape SL/S0, [m]

SL/S0	20	25	30	35	40	45	50	55	60	65	70
10.0	6.066	4.851	4.044	3.467	3.033	2.695	2.426	2.205	2.022	1.867	1.732
5.0	5.624	4.498	3.749	3.213	2.812	2.499	2.250	2.045	1.875	1.730	1.608
3.0	5.225	4.181	3.482	2.985	2.614	2.322	2.090	1.900	1.742	1.608	1.494
2.0	4.879	3.904	3.254	2.789	2.441	2.169	1.951	1.775	1.626	1.501	1.394
1.0	4.267	3.414	2.845	2.438	2.134	1.897	1.707	1.552	1.422	1.313	1.219
0.5	3.696	2.957	2.464	2.111	1.849	1.643	1.478	1.344	1.232	1.138	1.057
0.333	3.391	2.713	2.261	1.938	1.697	1.506	1.356	1.232	1.130	1.044	0.968
0.2	3.045	2.436	2.029	1.740	1.524	1.354	1.219	1.107	1.016	0.937	0.871
0.1	2.626	2.101	1.750	1.501	1.313	1.168	1.052	0.955	0.876	0.808	0.749

Using the tabulated values in table 4, we find an approximation to Leff

$$L_{eff}(\alpha, f_B) := \frac{15.562 \cdot \ln(\alpha) + 86.236}{f_B}$$

The relative error is given in table 5. We see that the error is small.

Table 5. The relative error for the Leff approximation

α	10	5	3	2	1	0.5	0.333	0.2	0.1
error, [%]	-0.6	1.1	1.1	0.6	-1.0	-2.0	-1.9	-0.5	4.2

The TL actual length is

$$L_{act}(\alpha, f_B, Q_{ts}) := L_{eff}(\alpha, f_B) - 0.6 \cdot \sqrt{\frac{S_L(\alpha, f_B, Q_{ts})}{\pi}}$$

The volume of the enclosure is

$$V_{TL}(\alpha, f_B, Q_{ts}) := \sigma_0(\alpha, f_B, Q_{ts}) \cdot S_d \cdot \frac{(1+\alpha)}{2} \cdot L_{act}(\alpha, f_B, Q_{ts})$$

Interesting relations

Rewriting this expression

$$\sigma_0(\alpha, f_B, Q_{ts}) := \rho \cdot c_{air} \cdot S_d \cdot D_Z(\alpha, f_B) \cdot D_{R2}(Q_{ts}) \cdot \frac{R_e}{BL^2}$$

Using fs = fB, we find

$$\sigma_0 := \rho \cdot c_{air} \cdot 0.0437 \cdot (-0.359 \cdot \ln(\alpha) + 0.9839) \cdot \frac{f_s \cdot S_d \cdot R_e}{Q_{ts}^{\,0.893} \cdot BL^2}$$

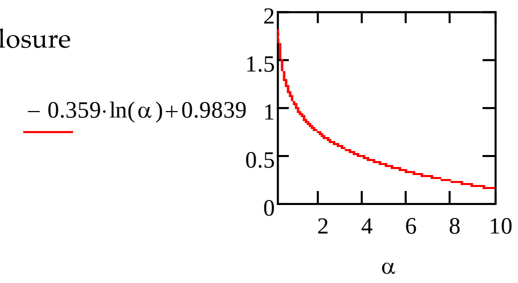
It has three parts:

1. A constant

$$\rho \cdot c_{air} \cdot 0.0437$$

2. One part that depends on the enclosure

$$(-0.359 \cdot \ln(\alpha) + 0.9839)$$



3. The driver

$$\frac{f_s \cdot S_d \cdot R_e}{Q_{ts}^{\,0.893} \cdot BL^2}$$

From the third part we deduce that drivers with low fs, high Qts and BL means a small enclosure.

For fs=90 Hz, Qts = 0.6, Re = 6.5 ohm, BL = 4.8 T/m, Sd=48 cm2, we get the following curves.